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THE FREE METABELIAN GROUP OF RANK TWO CONTAINS CONTINUOUSLY MANY NONISOMORPHIC SUBGROUPS

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ABSTRACT. It is proved that the free metabelian group of rank two contains continuously many nonisomorphic subgroups.

This short note gives an affirmative answer to the question—attributed to P. Hall—whether a finitely generated metabelian group contains continuously many nonisomorphic subgroups. This question came to the attention of the first author in the early seventies through J. E. Roseblade. To our knowledge it is still open. We resolve this question by proving the following

THEOREM. *Given a countable abelian group A there exists a subgroup U of the free metabelian group G of rank two, such that U_{ab} is the direct sum of A and a free abelian group of countable rank.*

PROOF. Let G be the free metabelian group on $\{x, y\}$. We consider the subgroup of G generated by G' and x. Clearly x generates an infinite cyclic group C. Let L denote G' as a ZC-module, where C acts by conjugation; it is easy to see that L is free of infinite rank. Now let A be any countable abelian group. We view A as a trivial ZC-module and write it as quotient of L. This gives rise to a short exact sequence

$$(\diamond) \qquad 0 \to K \to L \to A \to 0$$

of ZC-modules. We set $U = \mathrm{gp}(x, K)$. Then

$$U_{ab} = Z \oplus K/[K,U] = Z \oplus H_0(C,K).$$

In order to compute $H_0(C,K)$ we apply the functor $H_*(C,-)$ to the short exact sequence $(\diamond)$. We obtain

$$\cdots \to H_1(C,L) \to H_1(C,A) \to H_0(C,K) \to H_0(C,L) \to H_0(C,A) \to 0.$$

Since L is a free ZC-module, $H_1(C,L) = 0$ and $H_0(C,L)$ is free abelian. Moreover $H_1(C,A) = H_1(C,Z) \otimes A = A$. It follows that $H_0(C,K)$ is a direct sum of A and a free abelian group of countable rank. This proves the theorem.

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